the equation of the line through the two points (x_2, y_2) and (x_3, y_3) be written in determinantal form, the distance h from the point (x_1, y_1) to the line is

$$ h = \frac{\begin{vmatrix} x_1, y_1, 1 \\ x_2, y_2, 1 \\ x_3, y_3, 1 \end{vmatrix}}{\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}}. $$

Since the denominator is the base of the triangle determined by the three points, the determinantal form for the area will be evident. The transformation equations are as easily deduced by the direct application of Theorems I and II. The application of these three theorems to three dimensions is equally successful.

I do not at all mean to imply that these general proofs should be used to the exclusion of the time honored method of taking a figure in the first quadrant and deducing the result geometrically. These geometrical exercises may well be relegated to the problem lists. But the general method is the simpler, and the knowledge of the vector requisite for its use is not beyond the college freshman. For many students the vector idea can be introduced none too early. Anyone contemplating a new text on analytical geometry should certainly weigh these possibilities.

RECENT PUBLICATIONS.

REVIEWS.

MATHEMATICAL LOGIC.

A Survey of Symbolic Logic. By C. I. LEWIS. Berkeley, University of California Press. 1918. Royal 8vo. 6 + 409 pp. Price $4.00.

Molière's M. Jourdain was very much surprised when told that he had been using prose all his life. Equally astonished are many present-day mathematicians when informed that they have been using 'logical prose'—propositional functions, the Zermelo axiom, and the like—for a correspondingly long period.

What is this logical prose of which the mathematical and the logical world at large have been, till quite recently, so blissfully ignorant? It is the principles of modern deductive logic, known also as symbolic or mathematical logic. Though Professor Lewis prefers the term *symbolic*, Russell and his school seem to have established almost irrevocably the name *mathematical* logic. And Professor Lewis's book is a survey of the history of the various stages in the discovery of the principles of deductive logic.

What are these principles? Everyone has heard of the famous 'Laws of Thought'—the Laws of Identity, Contradiction, and Excluded Middle. Assuming that these laws, considered as principles of *logic* (not of thought), are necessary, are they also sufficient? Obviously not; for the principle of the Syllogism is just as necessary to logical procedure as are these laws. Are the four principles sufficient? How shall we decide? We can study the problem empirically.

We can make a careful and detailed investigation of some of the important proofs of mathematical theorems—for instance, in function theory—and after a step-by-step analysis we may find that these proofs employ various principles in addition to the four above mentioned.   Such is, for example, the principle that whenever we have three propositions p, q, and r, and we know that

(1) If p is true, then q implies r, we have a right to replace (1) by

(2) If p and q are both true, then r is true.   (This principle is called by Peano the principle of Importation, since it enables us to 'import' q into the hypothesis of the main implication.)   In mathematical proofs we can no more dispense with the principles of Importation than we can with the Syllogism or with the principle of Contradiction.   No more can we dispense with the 'obvious' principle that whenever any two propositions p and q are both true, then propositions q and p are both true, $i.e.$, the commutativity of the logical operation called 'propositional conjunction.'   And similarly with a host of other principles used implicitly by every logician and mathematician.

May not some of these principles be derivable from some of the others? Certainly.   The long train of researches, culminating in the mathematical logician's 'Bible'—Whitehead and Russell's volumes of *Principia Mathematica*— has demonstrated the fact that just as the entire system of euclidean geometry can be compressed deductively into a small number of unproved geometric propositions based on a small number of undefined geometric terms, so the set of propositions that constitute the system of deductive logic can be compressed into a surprisingly small number of fundamental logical propositions based on a small number of fundamental logical terms.

But the Rome of mathematical logic was not built in a day.   And the history of the long mathematical logical journey from Leibniz to Russell is a history of the triumph of the human intellect in every way as marvellous as the triumphs of the intellect in the realms of natural, as opposed to mental, experimentation.

The volume under review attempts to trace this history through its various stages.   In Chapter I Professor Lewis summarizes the development of mathematical logic from Leibniz through De Morgan, Boole, Jevons, and Peirce. In Chapter II he presents the formal principles of the algebra of logic, that is, the algebra as founded by Boole and improved by Schroeder.   This algebra, many of the laws of which are identical with those of number-algebras, has a 'function theory' that may well interest the student of mathematics from the standpoint of 'comparative algebra'.   In Chapter III we are introduced to the well-known interpretations of this Boole-Schroeder algebra, in terms of logical classes, propositions, and relations, and also, geometrically, in terms of planar regions.[1]   Ch. IV is devoted in part to an elementary and extremely successful

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[1] Although Whitehead (*A Treatise on Universal Algebra*, vol. I, p. 35) characterizes this algebra as "the only known member of the non-numerical genus of Universal Algebra," mathematical readers may be interested in the following purely numerical interpretation.   Let us call any positive integer, > 1, whose prime factors occur but once, a *Boolean* integer.   Examples: 6, 30, 70.   Choose any Boolean integer, B.   Let the elements, a, b, c, $\cdots$, of this algebra be the 2^n factors of B (including 1 and B).   Let $a \times b =$ H. C. F. (a, b).   Then $a + b =$ L. C. M. (a, b), "0" $= 1$, "1" $= B$, "$- a$" $= B \div a$, and $a \subset b = a$ is a factor of b.'

exposition of some of the main logical doctrines of the first volume of Whitehead and Russell's *Principia Mathematica*. In Chapter V Professor Lewis outlines a system of logic differing essentially from that of Russell in the meaning assigned to the fundamentally important relation of implication. Finally, in Chapter VI the leading views on mathematical and logical *method* are compared and contrasted. In the Appendix, Professor Lewis gives a translation, from Latin, of two fragments from Leibniz on the logic of classes.

The reviewer regrets that the author has omitted all mention of the various problems of postulational technique, as well as of *Mengenlehre*—problems intimately associated with the advance of mathematical logic. But in spite of these, perhaps inevitable, omissions, Professor Lewis, in presenting for the first time the connected history of the subject, has succeeded admirably.

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HARVARD UNIVERSITY.

College Algebra. By H. L. RIETZ and A. R. CRATHORNE. Revised Edition. New York, Henry Holt and Co., 1919. (First Edition, 1909.) 8vo. 14+268 pp. Price $1.60.

This new edition of the Rietz and Crathorne *College Algebra* differs from the first (*a*) in the addition of new exercises and the alteration of figures in old ones, (*b*) in the omission of some topics not in the main line of development, and (*c*) in a rewriting of certain sections which proved hard to teach in the original form.

The changes in the exercises are sufficient to undermine the utility of the sets of solutions which come into being on any campus when the same book has been used for several years. In some cases, in the interest of more careful grading, the order of examples has been changed and easy examples have been added. Other new examples introduce applications in fields not previously touched—in at least one case, involving 'war bread,' in a field which did not exist in 1909.

In the problems on the Theory of Equations, the authors have lost an excellent opportunity to fall in with the modern trend towards unification. Several problems (pp. 146–7) call for the depth to which a sphere of specified material will sink in water, but the necessary cubic equation is always given all 'set up.' If the student were asked to show that the depth to which a sphere of radius r and specific gravity ρ will sink is the root between 0 and $2r$ of the equation $x^3 - 3rx^2 + 4\rho r^3 = 0$, he would have to combine with the theory of equations a little solid geometry (the formula for a spherical segment), a fundamental physical definition, and a physical principle. This seems to be a striking instance of a useful bit of mathematics and physics which is not adequately treated in any course because it draws on physics and two separate mathematical subjects. Perhaps, moreover, the inclusion of this general equation would have helped the authors as well as the student, by making more likely the detection of the errors in the constant terms in Ex. 18 and 19 on page 146 in the revised edition. These errors really imply that the specific gravity of yellow pine is .9657 and of ice .693, although the first edition gives correct values, .657 and .93. It may be that this error arose from the desire not to ask (as the first edition did) for the determina-